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Non-Standard Physics in Leptonic and Semileptonic Decays of Charmed Mesons

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Recent measurements of the branching fraction for $D_s \to \ell\nu$ disagree with the Standard Model expectation, which relies on calculations of f_{D_s} from lattice QCD. This paper uses recent preliminary measurements from CLEO and a new preliminary lattice-QCD result from this conference to update the significance of the discrepancy. The "f_{D_s} puzzle" stands now at 3.5σ, with σ predominantly from the statistical uncertainty of the experiments. New physics scenarios that could solve the puzzle would also lead to non-Standard amplitudes mediating the semileptonic decays $D \to K\ell\nu$. This paper shows where the new amplitudes enter the differential rate and outlines where lattice QCD calculations are needed to confront recent and forthcoming measurements.

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1. Introduction

Recent years have witnessed significant improvements in charmed-meson leptonic and semileptonic decays, both in experimental measurements and in calculations of the hadronic transition amplitudes with lattice QCD. A puzzle has arisen, namely a discrepancy of approximately 3.5σ in the rate of the leptonic decay $D_s \to \ell\nu$, where ℓ is a muon or τ lepton [1]. If the measured counts have not fluctuated high, and the lattice QCD calculations are confirmed (by further calculations with 2+1 flavors of sea quarks), then this may be a signal of physics beyond the Standard Model [2].

If non-Standard interactions mediate $c\bar{s} \to \nu\bar{\ell}$, then they also alter, at some level, the rate and q^2-distribution of $D \to K\mu\nu$. ($D \to K\tau\nu$ is kinematically forbidden.) In this paper, section 2 recalls the origin of the leptonic discrepancy, incorporating new, preliminary results. Section 3 updates the new-physics analysis of Ref. [2] and extends it to encompass semileptonic decays. Then section 4 discusses the phenomenology of semileptonic decays in the context of new physics. For lattice QCD the main conclusion, discussed in section 5, is that precise calculations of the semileptonic form factors, including a tensor form factor defined below, are vital.

2. Leptonic Decays

In the Standard Model the partial width for $D_s \to \ell\nu_\ell$ is

$$\Gamma(D_s \to \ell\nu_\ell) = \frac{m_{D_s}}{8\pi} f_{D_s}^2 \left|G_F V_{cs}^* m_\ell\right|^2 \left(1 - m_\ell^2/m_{D_s}^2\right)^2, \tag{2.1}$$

where the decay constant f_{D_s} is defined by $\langle 0|\bar{s}\gamma^\mu\gamma_5 c|D_s(p)\rangle = i f_{D_s} p^\mu$, and is also computed via $(m_c + m_s)\langle 0|\bar{s}\gamma_5 c|D_s(p)\rangle = -i f_{D_s} m_{D_s}^2$; PCAC ensures that the two definitions are the same. The partial widths are small: for muonic decays owing to the helicity-suppression factor m_μ^2; for τ-leptonic decays owing to the phase-space factor $(1 - m_\tau^2/m_{D_s}^2)^2$. Experiments measure the branching fraction $B = \Gamma\tau_{D_s}$ but usually quote f_{D_s} assuming that no non-Standard amplitude contributes to Γ.

In this sense, f_{D_s} has been measured recently by the BaBar [3], Belle [4], and CLEO [5, 6] Collaborations. The experiments measure $B(D_s \to \ell\nu)$ directly, without complicated modeling of the events or background, and the experimental errors are principally statistical. Radiative corrections are at most 1–2%, and the discrepancy cannot be explained with any value of $|V_{cs}|$ consistent with a unitary $n \times n$ CKM matrix [2]. In summary, it seems sound to take the experimental measurements of f_{D_s} at face value, yielding Table 1. Treating both statistical and systematic uncertainties in quadrature, the average of the measurements in Table 1 is

$$f_{D_s}|_{\text{expt avg}} = 272 \pm 8 \text{ MeV} \tag{2.2}$$

combining $\mu\nu$ and $\tau\nu$ and including new results reported by CLEO at conferences through September 2008 [7]. Separate averages for the two final states are in Table 1.

Now let us turn to lattice QCD calculations of f_{D_s}. There are two calculations with 2+1 flavors of sea quarks, the first from the Fermilab Lattice and MILC Collaborations [8] and more recently from the HPQCD Collaboration [9]. These are

$$f_{D_s}|_{\text{HPQCD}} = 241 \pm 3 \text{ MeV}, \qquad f_{D_s}|_{\text{Fermilab-MILC}} = 249 \pm 11 \text{ MeV}, \tag{2.3}$$

where the Fermilab-MILC result is an update presented at this conference by Mackenzie [10]. Both calculations use the improved staggered Asqtad action for the sea quarks, taking advantage of the

Table 1: Recent experimental values of f_{D_s}. The preliminary update from CLEO can be found in Ref. [7].

final state	reference	f_{D_s} (MeV)	
		end 2007	2008 update
$\mu\nu$	BaBar [3]	$283 \pm 17 \pm 16$	
$\mu\nu$	CLEO [5]	$264 \pm 15 \pm 7$	$265.4 \pm 11.9 \pm 4.4$
$\mu\nu$	Belle [4]	$275 \pm 16 \pm 12$	
$\tau\nu$ ($\tau \to \pi\nu$)	CLEO [5]	$310 \pm 25 \pm 8$	$271 \pm 20 \pm 4$
$\tau\nu$ ($\tau \to e\nu\bar{\nu}$)	CLEO [6]	$273 \pm 16 \pm 8$	
$\mu\nu$	our average	273 ± 11	271 ± 10
$\tau\nu$	our average	285 ± 15	272 ± 13

freely available MILC ensembles [11]. In the following, I use the average of the two results in Eq. (2.3) with no correlation, because the dominant systematic errors differ.

The experimental average of f_{D_s} lies 12.5% above that of lattice QCD, and the significance of the discrepancy is

$$3.5\sigma = 2.9\sigma \oplus 2.2\sigma, \tag{2.4}$$

where the two entries on the right-hand side are for $\mu\nu$ and $\tau\nu$ separately. Before the update from CLEO [7] the discrepancy was $3.8\sigma = 2.7\sigma \oplus 2.9\sigma$ [2]. Reference [1] omits BaBar's result and reports 3.4σ, and with CLEO's update this approach yields 3.2σ. One should bear in mind that the yardstick for σ is the experimental *statistical* error: Were one to double HPQCD's error (without justification), the total discrepancy would remain 3.0σ. Indeed, the same methods agree with experiment for f_π, f_K, charmonium mass splittings, and especially m_{D_s}, m_{D^+}, and f_{D^+} [9, 12].

3. New Physics

If the discrepancy cannot be traced to a fluctuation or error in either the measurements or the calculation(s), then one should turn to non-Standard physics as an explanation. The new particles must be heavy to have escaped direct detection, so one may consider an effective Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{eff}} &= M^{-2}C_A^\ell(\bar{s}\gamma^\mu\gamma_5 c)(\bar{\nu}_{\ell L}\gamma_\mu \ell_L) + M^{-2}C_P^\ell(\bar{s}\gamma_5 c)(\bar{\nu}_{\ell L}\ell_R) - M^{-2}C_V^\ell(\bar{s}\gamma^\mu c)(\bar{\nu}_{\ell L}\gamma_\mu \ell_L) \\ &+ M^{-2}C_S^\ell(\bar{s}c)(\bar{\nu}_{\ell L}\ell_R) + M^{-2}C_T^\ell(\bar{s}\sigma^{\mu\nu}c)(\bar{\nu}_{\ell L}\sigma_{\mu\nu}\ell_R) + \text{H.c.},\end{aligned} \tag{3.1}$$

where M is a high mass scale. This $\mathscr{L}_{\text{eff}}$ extends the effective Lagrangian of Ref. [2] to include interactions that mediate $D \to K\ell\nu$. The experiments do not identify the neutrino flavor or helicity, but Eq. (3.1) assumes $\bar{\nu}_{\ell L}$ to be of lepton flavor ℓ and omits right-handed neutrinos. In this way the resulting non-Standard amplitudes can interfere with the Standard W-mediated amplitude and explain the sought-after effect of 10–15% (in the amplitude).

These effective interactions change the rate for leptonic decay, by substituting in Eq. (2.1)

$$G_F V_{cs}^* m_\ell \to G_F V_{cs}^* m_\ell + \frac{C_A^\ell}{\sqrt{2}M^2} m_\ell + \frac{C_P^\ell}{\sqrt{2}M^2}\frac{m_{D_s}^2}{m_c + m_s}. \tag{3.2}$$

Because (conventionally) V_{cs} is real, one sees that one or both of C_A^ℓ, C_P^ℓ must have a positive real part. If only one of these reduces the discrepancy to 1σ, one can derive the bounds

$$M(\mathrm{Re}C_A^\ell)^{-1/2} \lesssim 855~\mathrm{GeV}, \qquad M(\mathrm{Re}C_P^\ell)^{-1/2} \lesssim 1070~\mathrm{GeV}\sqrt{m_\tau/m_\ell}, \tag{3.3}$$

updating Ref. [2] to reflect CLEO's new preliminary measurements and treating the $\mu\nu$ and $\tau\nu$ discrepancies as a single effect.

The effective Lagrangian can arise from the tree-level exchange of non-Standard particles, in which case M is simply the new particle's mass. Reference [2] found a few possibilities. One is the s-channel annihilation through a charged Higgs boson, in a new model designed so that the Yukawa couplings satisfy $y_s \ll y_c$ and $y_c, y_\tau \sim 1$. But this model also has $y_d < y_s$, thereby predicting a 10–15% deviation in the amplitude for $D^+ \to \ell^+\nu$. This is now disfavored, because CLEO's new measurement of f_{D^+} [13] agrees perfectly with lattice QCD [8, 9, 10]. Another candidate is the t-channel exchange of a charge $+\frac{2}{3}$ leptoquark, which can arise in various ways, all of which are disfavored by non-observation of $\tau \to \mu s\bar{s}$. The most promising mechanism is the u-channel exchange of an SU(2)-singlet, charge $-\frac{1}{3}$ leptoquark, namely a particle with the quantum numbers as a down-type scalar quark $\tilde{d}$ in supersymmetric models. It couples via the R-violating Lagrangian

$$\mathscr{L}_{\mathrm{LQ}} = \kappa_{2\ell}\,(\bar{c}_L\ell_L^c - \bar{s}_L\nu_{\ell L}^c)\,\tilde{d} + \kappa'_{2\ell}\,\bar{c}_R\ell_R^c\tilde{d} + \mathrm{H.c.}, \tag{3.4}$$

where the superscript c denotes charge conjugation, and $\kappa_{2\ell}$ and $\kappa'_{2\ell}$ are complex parameters (in general, entries of Yukawa matrices). When $M = m_{\tilde{d}} \gg m_{D_s}$ one can derive $\mathscr{L}_{\mathrm{eff}}$ with

$$C_A^\ell = C_V^\ell = \tfrac{1}{4}|\kappa_{2\ell}|^2, \qquad C_P^\ell = C_S^\ell = \tfrac{1}{4}\kappa_{2\ell}\kappa'^*_{2\ell} = -2C_T^\ell. \tag{3.5}$$

If $\kappa_{2\ell}$ is independent of ℓ and either $\kappa'_{2\ell} \propto m_\ell$ or $|\kappa'_{2\ell}/\kappa_{2\ell}| \ll m_\ell m_c/m_{D_s}^2$, then these interactions could explain why the discrepancy appears in both $\mu\nu$ and $\tau\nu$ channels. Generalizations of Eq. (3.4) appear in non-Standard models that modify the interference phase of B_s^0-$\bar{B}_s^0$ [14], explain quark masses [15], induce deviations in $B_{(c)}^+ \to \ell\nu$ [16], generate neutrino masses [17], or enhance rare D decays [18].

4. Semileptonic Decays

To obtain further information about a possible non-Standard cause of the effective $\bar{s}c\bar{\nu}\ell$ vertex, one can turn to other processes. One would be the production charmed quarks in neutrino scattering off strange sea quarks in nucleons. Another set consists of the semileptonic decays $D^0 \to K^-\mu^+\nu_\mu$, $D^+ \to \bar{K}^0\mu^+\nu_\mu$, and their charge conjugates. A full understanding of these decays will require lattice QCD calculations of the hadronic transition.

Let us start by reviewing the kinematics of three-body decays. Let the D-meson, kaon, lepton, and neutrino 4-momenta be denoted p, k, ℓ, and ν. There are two Lorentz independent invariants, which may be taken to be $E_\ell = p\cdot\ell/m_D$ and $E_K = p\cdot k/m_D$, namely the lepton and kaon energies in the D meson's rest frame. Often instead of E_K the mass-squared of the leptonic system is used, $q^2 = m_D^2 + m_K^2 - 2m_DE_K$, $q = \ell + \nu = p - k$. For brevity the formulae given below use both E_K and q^2. The kinematically allowed region is shown in the Dalitz plot, Fig. 1. The discussion given below is somewhat simpler with the variable

$$E_{\ell\perp} = \frac{p\cdot\ell}{m_D} - \frac{p\cdot q q\cdot\ell}{m_D q^2} = E_\ell - \tfrac{1}{2}(m_D - E_K)\left(1 + m_\ell^2/q^2\right), \tag{4.1}$$

and the allowed region, for fixed E_K is $-E_{\ell\perp}^{\max} \leq E_{\ell\perp} \leq E_{\ell\perp}^{\max}$, $E_{\ell\perp}^{\max} = \frac{1}{2}(1-m_\ell^2/q^2)\sqrt{E_K^2-m_K^2}$. The allowed region for E_K is $m_K \leq E_K \leq (m_D^2+m_K^2-m_\ell^2)/2m_D$, or $m_\ell^2 \leq q^2 \leq (m_D-m_K)^2$.

The doubly-differential rate for $D \to K\ell\nu$ is

$$\begin{aligned}
\frac{d^2\Gamma}{dE_K\,dE_{\ell\perp}} &= \frac{m_D}{(2\pi)^3}\left\{ \left[\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)-4E_{\ell\perp}^2\right]\left|G_FV_{cs}^*+G_V^\ell\right|^2\left|f_+(q^2)\right|^2 \right.\\
&+ \frac{q^2-m_\ell^2}{4m_D^2}\left|m_\ell\left(G_FV_{cs}^*+G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2}+G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right|^2\left|f_0(q^2)\right|^2 \\
&+ \left[\frac{m_\ell^2}{4m_D^2}\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)+\frac{4q^2}{m_D^2}E_{\ell\perp}^2\right]\left|G_T^\ell\right|^2\left|f_2(q^2)\right|^2 \\
&- \frac{2m_\ell}{m_D}\left(E_K^2-m_K^2\right)\left(1-m_\ell^2/q^2\right)\mathrm{Re}\left[\left(G_FV_{cs}^*+G_V^\ell\right)G_T^{\ell*}f_+(q^2)f_2^*(q^2)\right] \\
&- \frac{2m_\ell}{m_D}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\left(G_FV_{cs}^*+G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2}+G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)\times\right.\\
&\qquad\qquad \left.\left(G_FV_{cs}+G_V^{\ell*}\right)f_0(q^2)f_+^*(q^2)\right] \\
&\left.+ \frac{2q^2}{m_D^2}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\left(G_FV_{cs}^*+G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2}+G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)G_T^{\ell*}f_0(q^2)f_2^*(q^2)\right]\right\},
\end{aligned} \tag{4.2}$$

where $G_{V,S,T}^\ell = C_{V,S,T}^\ell/\sqrt{2}M^2$, and the form factors f_+, f_0, and f_2 are defined via

$$\langle K(k)|\bar{s}\gamma^\mu c|D(p)\rangle = \left(p^\mu+k^\mu-\frac{m_D^2-m_K^2}{q^2}q^\mu\right)f_+(q^2)+\frac{m_D^2-m_K^2}{q^2}q^\mu f_0(q^2), \tag{4.3}$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu}c|D(p)\rangle = im_D^{-1}(p^\mu k^\nu-p^\nu k^\mu)f_2(q^2), \tag{4.4}$$

$$\langle K(k)|\bar{s}c|D(p)\rangle = \frac{m_D^2-m_K^2}{m_c-m_s}f_0(q^2), \tag{4.5}$$

and f_0 appears for both the vector and scalar currents owing to CVC. Integrating over lepton energy

$$\begin{aligned}
\frac{d\Gamma}{dE_K} &= \frac{m_D}{(2\pi)^3}\sqrt{E_K^2-m_K^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left\{(E_K^2-m_K^2)\frac{2q^2+m_\ell^2}{3q^2}\left|G_FV_{cs}^*+G_V^\ell\right|^2\left|f_+(q^2)\right|^2\right.\\
&+ \frac{q^2}{4m_D^2}\left|m_\ell\left(G_FV_{cs}^*+G_V^\ell\right)\frac{m_D^2-m_K^2}{q^2}+G_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right|^2\left|f_0(q^2)\right|^2 \\
&+ (E_K^2-m_K^2)\frac{q^2+2m_\ell^2}{3m_D^2}\left|G_T^\ell\right|^2\left|f_2(q^2)\right|^2 \\
&\left.- 2\frac{m_\ell}{m_D}(E_K^2-m_K^2)\,\mathrm{Re}\left[\left(G_FV_{cs}^*+G_V^\ell\right)G_T^{\ell*}f_+(q^2)f_2^*(q^2)\right]\right\}.
\end{aligned} \tag{4.6}$$

Note that two terms with interference between form factors vanish after integration: the variable $E_{\ell\perp}$ renders this feature especially transparent. We shall use these formulae to diagnose how new interactions mediating $D_s \to \ell\nu$ would alter the semileptonic rate and differential distributions.

In many non-Standard models, including those with the charge $-\frac{1}{3}$ leptoquark $\tilde{d}$, one finds $C_V^\ell = C_A^\ell$ and $C_S^\ell = C_P^\ell$ (cf. Eq. (3.5)). If the f_{D_s} puzzle is solved by the C_A^ℓ interaction, then the C_V^ℓ interaction generates a similarly large enhancement in the semileptonic rate. This consequence is easily seen from the $|f_+|^2$ contribution to the rate: the other Standard contribution, with $|f_0|^2$, is

suppressed by $(m_\ell/m_D)^2$, which is 3×10^{-3} (7×10^{-8}) for μ (e). On the other hand, if the f_{D_s} puzzle is solved by the C_P^ℓ interaction, then it will be difficult to observe the companion semileptonic contribution. To enhance both $D_s\to\tau\nu$ and $D_s\to\mu\nu$, some mechanism should lead to $C_P^\ell\propto m_\ell$, and then (in the leptoquark example) $C_S^\ell\propto m_\ell$, $C_T^\ell\propto m_\ell$ also. In that case, the non-Standard contributions are small corrections to a suppressed contribution.

The doubly-differential rate suggests a challenging way to observe the effects of non-vanishing C_S^ℓ and C_T^ℓ. In the asymmetry

$$\mathscr{A}_\perp=\frac{\Gamma(E_{\ell\perp}>0)-\Gamma(E_{\ell\perp}<0)}{\Gamma(E_{\ell\perp}>0)+\Gamma(E_{\ell\perp}<0)}=\frac{N(E_{\ell\perp}>0)-N(E_{\ell\perp}<0)}{N(E_{\ell\perp}>0)+N(E_{\ell\perp}<0)} \tag{4.7}$$

everything but the last two lines of Eq. (4.2) cancels. To obtain a 7% measurement of $\mathscr{A}_\perp$, one would need around 10^7 semimuonic events in each half of the modified Dalitz plane. One can generalize this asymmetry to any region in the E_K-$E_{\ell\perp}$ plane that is symmetric about $E_{\ell\perp}=0$, or to any moment of the distribution odd in $E_{\ell\perp}$. For example, when the kaon momentum is low, phase space naturally suppresses the $|f_+|^2$ contribution, perhaps helpfully.

5. Conclusions

From Eqs. (4.2) and (4.6) one sees that the first concern of future lattice calculations is to improve on the 7% uncertainty of the only 2+1 flavor calculation of $f_+(q^2)$ [19]. With current semielectronic measurements [20, 21], one could test for new contributions to the $\bar{s}c\bar{\nu}_e e$ vertex (for which there is not yet any evidence). Semimuonic measurements are needed for a direct test of the f_{D_s} puzzle. Once the event yields become high enough to measure $\mathscr{A}_\perp$, it will be necessary to have accurate calculations of the scalar and tensor form factors, f_0 and f_2. CLEO and the B factories have somewhat more data to analyze, and BES-III should record thousands of events [22].

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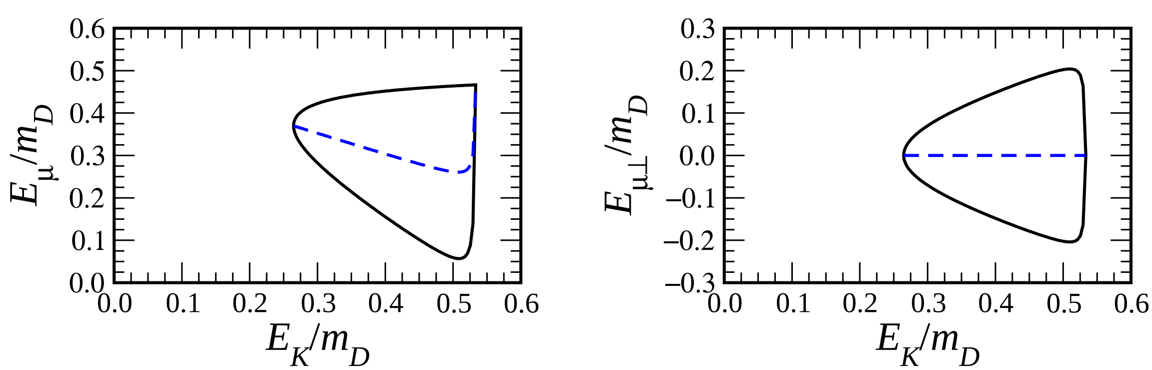

Figure 1: Dalitz plots for $D\to K\ell\nu$, with $m_\ell=m_\mu$. The left panel shows E_ℓ, the lepton energy in the D meson's rest frame; the right panel shows $E_{\ell\perp}$, defined in Eq. (4.1). The dashed (blue) lines show $E_{\ell\perp}=0$.

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A. All Semileptonic Formulas

To treat the SM and NP efficiently, we shall now write the effective Lagrangian as

$$\begin{aligned}\mathscr{L}_{\text{eff}} &= M^{-2}\bar{C}_A^\ell(\bar{s}\gamma^\mu\gamma_5 c)(\bar{\nu}_L\gamma_\mu\ell_L) - M^{-2}\bar{C}_V^\ell(\bar{s}\gamma^\mu c)(\bar{\nu}_L\gamma_\mu\ell_L) + M^{-2}C_P^\ell(\bar{s}\gamma_5 c)(\bar{\nu}_L\ell_R) \\ &+ M^{-2}C_S^\ell(\bar{s}c)(\bar{\nu}_L\ell_R) + M^{-2}C_T^\ell(\bar{s}\sigma^{\mu\nu}c)(\bar{\nu}_L\sigma_{\mu\nu}\ell_R) + \text{H.c.},\end{aligned} \tag{A.1}$$

where

$$\bar{C}_{V,A}^\ell = \sqrt{2}M^2 G_F V_{cs}^* + C_{V,A}^\ell, \qquad \frac{\bar{C}_{V,A}^\ell}{\sqrt{2}M^2} = G_F V_{cs}^* + \frac{C_{V,A}^\ell}{\sqrt{2}M^2}. \tag{A.2}$$

This $\mathscr{L}_{\text{eff}}$ mediates $D \to K\ell\nu$. (By analogy, one can extend this to the semileptonic decay of any pseudoscalar meson.) Let the D-meson, kaon, lepton, and neutrino 4-momenta be denoted p, k, ℓ, and ν, as above. The amplitude is

$$\begin{aligned}\langle \ell\nu K|i\mathscr{L}_{\text{eff}}|D\rangle = iM^{-2}\bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)\Big[C_S^\ell\langle K|\bar{s}c|D\rangle &- \bar{C}_V^\ell\gamma_\mu\langle K|\bar{s}\gamma^\mu c|D\rangle \\ &+ C_T^\ell\sigma_{\mu\nu}\langle K|\bar{s}\sigma^{\mu\nu}c|D\rangle\Big]v(\ell).\end{aligned} \tag{A.3}$$

The hadronic matrix elements are re-expressed as form factors in Eqs. (4.3)–(4.4). For the leptons we require the spinor combinations

$$\bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)(\not{p}+\not{k})_\perp v(\ell) = 2\bar{u}(\nu)\not{p}_\perp\tfrac{1}{2}(1-\gamma_5)v(\ell), \tag{A.4}$$

$$\bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)\not{q}v(\ell) = -m_\ell\bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)v(\ell), \tag{A.5}$$

$$(p^\mu k^\nu - p^\nu k^\mu)\bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)i\sigma_{\mu\nu}v(\ell) = -2\bar{u}(\nu)\not{p}_\perp\not{q}\tfrac{1}{2}(1+\gamma_5)v(\ell), \tag{A.6}$$

where for any four-vector r, $r_\perp^\mu = r^\mu - (r\cdot q/q^2)q^\mu$. Inserting Eqs. (4.3)–(4.4) and (A.4)–(A.6) into Eq. (A.3),

$$\begin{aligned}\langle \ell\nu K|i\mathscr{L}_{\text{eff}}|D\rangle = iM^{-2}\Bigg\{&\left(m_\ell\bar{C}_V^\ell\frac{m_D^2-m_K^2}{q^2} + C_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)\bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)v(\ell)\,f_0(q^2) \\ &- 2\bar{C}_V^\ell\bar{u}(\nu)\not{p}_\perp\tfrac{1}{2}(1-\gamma_5)v(\ell)\,f_+(q^2) - 2C_T^\ell m_D^{-1}\bar{u}(\nu)\not{p}_\perp\not{q}\tfrac{1}{2}(1+\gamma_5)v(\ell)\,f_2(q^2)\Bigg\}.\end{aligned} \tag{A.7}$$

The differential rate is (see PDG)

$$\frac{d^2\Gamma}{dE_K\,dE_\ell} = \frac{1}{(2\pi)^3}\frac{1}{8m_D}\sum_{\text{spins}}|\langle\ell\nu K|i\mathscr{L}_{\text{eff}}|D\rangle|^2, \tag{A.8}$$

where $E_K = p\cdot k/m_D$ and $E_\ell = p\cdot\ell/m_D$ are the energies of the kaon and lepton in the rest frame of the D meson. To sum over lepton and neutrino polarization states, we need

$$\sum_{\text{spins}}\bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)v(\ell)\bar{v}(\ell)\tfrac{1}{2}(1-\gamma_5)u(\nu) = 2\nu\cdot\ell = q^2 - m_\ell^2, \tag{A.9}$$

$$\sum_{\text{spins}}\bar{u}(\nu)\not{p}_\perp\tfrac{1}{2}(1-\gamma_5)v(\ell)\bar{v}(\ell)\tfrac{1}{2}(1+\gamma_5)\not{p}_\perp u(\nu) = -p_\perp^2(q^2-m_\ell^2) - (2p_\perp\cdot\ell)^2, \tag{A.10}$$

$$\sum_{\text{spins}}\bar{u}(\nu)\not{p}_\perp\not{q}\tfrac{1}{2}(1+\gamma_5)v(\ell)\bar{v}(\ell)\tfrac{1}{2}(1-\gamma_5)\not{q}\not{p}_\perp u(\nu) = q^2(2p_\perp\cdot\ell)^2 - m_\ell^2 p_\perp^2(q^2-m_\ell^2), \tag{A.11}$$

$$\sum_{\text{spins}} \bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)v(\ell)\bar{v}(\ell)\tfrac{1}{2}(1+\gamma_5)\not{p}_\perp u(\nu) \;=\; 2m_\ell p_\perp\cdot\ell, \tag{A.12}$$

$$\sum_{\text{spins}} \bar{u}(\nu)\tfrac{1}{2}(1+\gamma_5)v(\ell)\bar{v}(\ell)\tfrac{1}{2}(1-\gamma_5)\not{q}\not{p}_\perp u(\nu) \;=\; -2q^2 p_\perp\cdot\ell, \tag{A.13}$$

$$\sum_{\text{spins}} \bar{u}(\nu)\not{p}_\perp\tfrac{1}{2}(1-\gamma_5)v(\ell)\bar{v}(\ell)\tfrac{1}{2}(1-\gamma_5)\not{q}\not{p}_\perp u(\nu) \;=\; m_\ell p_\perp^2(q^2-m_\ell^2), \tag{A.14}$$

in which $p_\perp^2 = -(E_K^2-m_K^2)m_D^2/q^2$. Note that $E_K^2-m_K^2$ is nothing but the kaon's squared three-momentum in the D-meson rest frame. It is convenient to change from the variable E_ℓ to $E_{\ell\perp} = p_\perp\cdot\ell/m_D$, defined in the text. With this variable, it is easy to see which contributions vanish after integrating over lepton energy.

The doubly-differential rate is then

$$\begin{aligned}
\frac{d^2\Gamma}{dE_K\,dE_{\ell\perp}} \;=\; & \frac{1}{(2\pi)^3}\frac{m_D}{2M^4}\left\{\left[(E_K^2-m_K^2)\frac{q^2-m_\ell^2}{q^2}-4E_{\ell\perp}^2\right]\left|\bar{C}_V^\ell\right|^2|f_+(q^2)|^2\right.\\
&+\;\frac{q^2-m_\ell^2}{4m_D^2}\left|m_\ell\bar{C}_V^\ell\frac{m_D^2-m_K^2}{q^2}+C_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right|^2|f_0(q^2)|^2\\
&+\;\left[\frac{m_\ell^2}{m_D^2}(E_K^2-m_K^2)\frac{q^2-m_\ell^2}{q^2}+\frac{4q^2}{m_D^2}E_{\ell\perp}^2\right]\left|C_T^\ell\right|^2|f_2(q^2)|^2\\
&-\;\frac{2m_\ell}{m_D}(E_K^2-m_K^2)\frac{q^2-m_\ell^2}{q^2}\,\mathrm{Re}\left[\bar{C}_V^\ell C_T^{\ell\,*}f_+(q^2)\,f_2^*(q^2)\right]\\
&-\;\frac{2m_\ell}{m_D}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\bar{C}_V^\ell\frac{m_D^2-m_K^2}{q^2}+C_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)\bar{C}_V^{\ell *}f_0(q^2)\,f_+^*(q^2)\right]\\
&+\;\left.\frac{2q^2}{m_D^2}E_{\ell\perp}\,\mathrm{Re}\left[\left(m_\ell\bar{C}_V^\ell\frac{m_D^2-m_K^2}{q^2}+C_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right)C_T^{\ell\,*}f_0(q^2)\,f_2^*(q^2)\right]\right\}.
\end{aligned}\tag{A.15}$$

The singly-differential rate is

$$\begin{aligned}
\frac{d\Gamma}{dE_K} \;=\; & \frac{1}{(2\pi)^3}\frac{m_D}{2M^4}\sqrt{E_K^2-m_K^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left\{(E_K^2-m_K^2)\frac{2q^2+m_\ell^2}{3q^2}\left|\bar{C}_V^\ell\right|^2|f_+(q^2)|^2\right.\\
&+\;\frac{q^2}{4m_D^2}\left|m_\ell\bar{C}_V^\ell\frac{m_D^2-m_K^2}{q^2}+C_S^\ell\frac{m_D^2-m_K^2}{m_c-m_s}\right|^2|f_0(q^2)|^2\\
&+\;\left.(E_K^2-m_K^2)\frac{q^2+2m_\ell^2}{3m_D^2}\left|C_T^\ell\right|^2|f_2(q^2)|^2-\frac{2m_\ell}{m_D}(E_K^2-m_K^2)\,\mathrm{Re}\left[\bar{C}_V^\ell C_T^{\ell\,*}f_+(q^2)\,f_2^*(q^2)\right]\right\}
\end{aligned}\tag{A.16}$$

$$\begin{aligned}
=\; & \frac{m_D}{(2\pi)^3}\sqrt{E_K^2-m_K^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left\{(E_K^2-m_K^2)\frac{2q^2+m_\ell^2}{3q^2}\left|G_FV_{cs}^*+\frac{C_V^\ell}{\sqrt{2}M^2}\right|^2|f_+(q^2)|^2\right.\\
&+\;\frac{q^2}{4m_D^2}\left|m_\ell\left(G_FV_{cs}^*+\frac{C_V^\ell}{\sqrt{2}M^2}\right)\frac{m_D^2-m_K^2}{q^2}+\frac{C_S^\ell}{\sqrt{2}M^2}\frac{m_D^2-m_K^2}{m_c-m_s}\right|^2|f_0(q^2)|^2\\
&+\;(E_K^2-m_K^2)\frac{q^2+2m_\ell^2}{3m_D^2}\left|\frac{C_T^\ell}{\sqrt{2}M^2}\right|^2|f_2(q^2)|^2\\
&-\;\left.2\frac{m_\ell}{m_D}(E_K^2-m_K^2)\,\mathrm{Re}\left[\left(G_FV_{cs}^*+\frac{C_V^\ell}{\sqrt{2}M^2}\right)\frac{C_T^{\ell\,*}}{\sqrt{2}M^2}f_+(q^2)\,f_2^*(q^2)\right]\right\}.
\end{aligned}\tag{A.17}$$